

# Constrained Texture Mapping for Polygonal Meshes

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## Overview

- Introduction
- Notion of Parameterization
- Choosing a Criterion
- Least-squares Constraints
- Applications



## 1 Introduction



interactive features specification



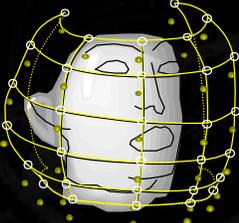
## 1 Introduction



Cylindrical map



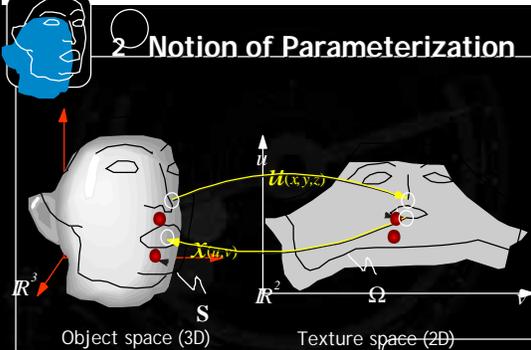
## 1 Introduction



Indirect manipulation:  
Projection on a deformable surface



## 2 Notion of Parameterization



Object space (3D)  $\mathbb{R}^3$   $S$

Texture space (2D)  $\mathbb{R}^2$   $\Omega$

$u(x,y,z)$



## 2 Notion of Parameterization

$\mathbb{R}^3$   $\mathbb{R}^2$

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## 2 Notion of Parameterization

$(u, v) = T(u_p, v_p)$   $(x, y, z) = X(u, v)$

$(x, y, z) = X \circ T(u_p, v_p) = X \circ T(u, v)$

[Beier], [Guentert], [Pighin]

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## 2 Notion of Parameterization

Minimize  $C(u_1, v_1, u_2, v_2, \dots, u_n, v_n)$

- **Harmonic maps** [Eells], [Eck]
- **Barycentric maps** [Tutte], [Floater], [Levy]
- **Non-Linear criteria** :
  - Deformation energy [Maillot]
  - MIPS [Hormann]
  - Texture Mapping PMs [Sander]

➔ New criterion

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## 2 Notion of Parameterization

Minimize  $C(u_1, v_1, u_2, v_2, u_3, v_3, \dots, u_n, v_n)$

$\mathbb{R}^3$   $\mathbb{R}^2$

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## 3 Choosing a Criterion

Harmonic map :  $C = \sum_{(i,j) \in E} k_{i,j} ((u_i - u_j)^2 + (v_i - v_j)^2)$

Fixed border

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## 3 Choosing a Criterion

Conformal map :  $C = \sum_{T \in T} \|\text{grad}(u|T) - i \cdot \text{grad}(v|T)\|^2$

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### 3 Choosing a Criterion

[Eckstein] Adding Steiner points in the triangulation




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### 3 Choosing a Criterion

Suitable properties for the criterion C:

- Good behavior around constraints
- Extrapolating capabilities

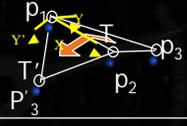
Our criterion:



$$\Delta_{\text{reg}}^2(T, T') = \|\text{grad}(u|_T) - \text{grad}(u|_{T'})\|^2 + \|\text{grad}(v|_T) - \text{grad}(v|_{T'})\|^2$$

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### 3 Choosing a Criterion

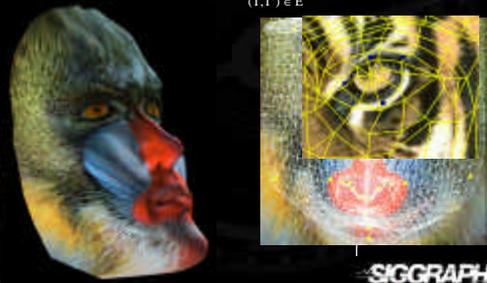
$$\text{grad } v(T) = \begin{bmatrix} \sum_{l=i,j,k} TX_l v_l \\ \sum_{l=i,j,k} TY_l v_l \end{bmatrix}$$


$$\Delta_{\text{reg}}^2(T, T') = (u_1(TY_1 - TY'_1) + u_2(TY_2 - TY'_2) + u_3 \cdot TY_3 + u'_3 \cdot TY'_3)^2 + (v_1(TY_1 - TY'_1) + v_2(TY_2 - TY'_2) + v_3 \cdot TY_3 + v'_3 \cdot TY'_3)^2$$

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### 3 Choosing a Criterion

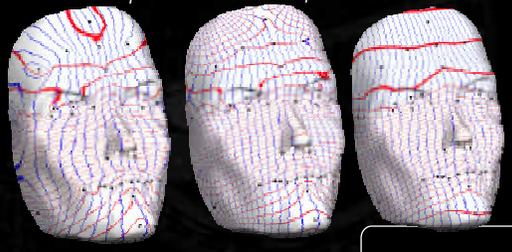
Our criterion :  $C = \sum_{(T, T') \in \mathcal{P}^g} \Delta_{\text{reg}}^2(T, T')$



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### 3 Choosing a Criterion

Harmonic map      Conformal map      Our criterion



Iso-parametric curves

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### 3 Choosing a Criterion

CONSTRAINED  
TEXTURE  
MAPPING

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### 3 Choosing a Criterion

$c = 1/d \cdot \sum p_i$   
 $(u_c, v_c) = 1/d \cdot (\sum u_i, \sum v_i)$

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### 4 Least-Squares Constraints

Using arbitrary points (not only vertices)

$\Delta_{fit}^2(p_i) = \{U(p_i) - (u_p, v_p)\}^2$

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### 4 Least-Squares Constraints

$\Delta_{fit}^2(p_i) = \{ \lambda_1 \cdot (u_i, v_i) + \lambda_2 \cdot (u_j, v_j) + \lambda_3 \cdot (u_k, v_k) - (u_p, v_p) \}^2$

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### 4 Least Squares Constraints

$$C = \underbrace{\sum_i \Delta_{fit}^2(p_i)}_{\text{Fitting}} + \underbrace{\epsilon \sum_{(T,T') \in E} \Delta_{reg}^2(T, T')}_{\text{Regularization}}$$

Both least-squares and hard constraints can be used at the same time.

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### 4 Least Squares Constraints

Time (s)	Precond. Conj. Grad	Conj. Grad	S.O.R.
0	10	10	10
5	~3	~4	~5
10	~2	~3	~4
15	~1.5	~2.5	~3.5
20	~1.2	~2.2	~3.2

10k triangles, 16hz, PIII

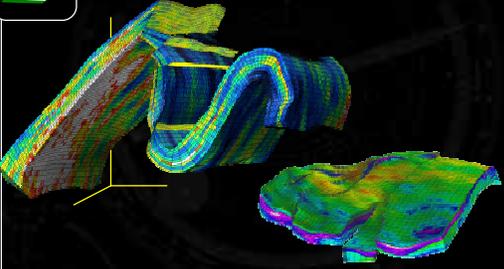
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### 5 Applications

- 3D Paint Systems [Pedersen]
- Short movie « Face it »

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 5 Applications



Constrained 3D gridding



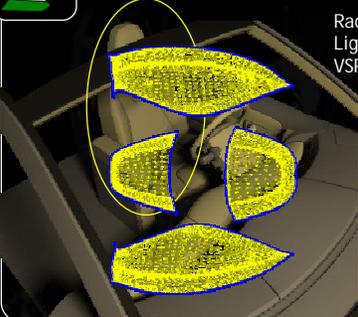
 5 Applications



Oil&Gas industry  
Gocad consortium  
T-Surf Corp.



 5 Applications



Radiosity  
Light maps generation  
VSP-Tech



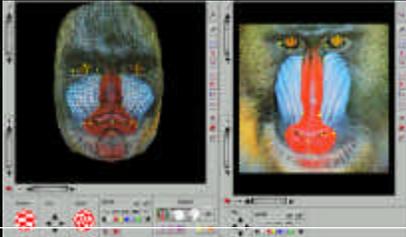
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 Try it at the Creative Applications Lab !

Graphite: experimental 3D modeler



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 Questions ?

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